Test Corrections

- due (last submission) by Oct 18
- work by yourself and/or with me.

① For any problem(s) where points
all
were missed: on a separate piece of paper – redo the problem completely.
② In sentences, explain the math mistake(s) made in the original solution.

If both ① & ② are correct, I will give back $\frac{1}{2}$ of the points.

- You can resubmit as many times as needed before the 10/18 deadline.

Example Suppose Austin throws a baseball with initial velocity 75 miles/hr, at an angle of 30° from the horizontal, starting at a height of 5.0 ft.

1. When does the ball reach its maximum height?
2. What is the maximum height?
3. When does the ball hit the ground?
4. How far did the ball go?

(Last times) ....

(vertical velocity) $s'(t) = -32t + 55$

$\Rightarrow s(t) = -16t^2 + 55t + 5$ vertical position

(b) max height $= s(1.71) = -16(1.71)^2 + 55(1.71) + 5.0$

$= 52.3$ ft

ft/s² s ft/s s ft

(c) What is t when $s(t) = 0$?

$-16t^2 + 55t + 5 = 0$

$$\Rightarrow t = \frac{-55 \pm \sqrt{(55)^2 - 4\cdot(-16)(5)}}{2\cdot(-16)}$$

$\boxed{t = 3.53\,s}$ ← hits the ground.

(d) How far did the ball go?

Horizontal velocity $= 95.3$ ft/s

Horizontal position when it hits the ground

$= (95.3\,ft/s)(3.53\,s) \approx \boxed{336.4\,ft}$

Additional question:

(e) How fast was the ball going when it hit the ground?

vertical velocity $= s'(t) = -32(3.53) + 55$

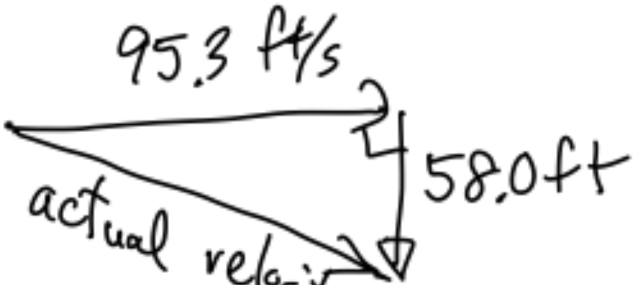
$t = 3.53$

$= 57.96 \approx 58.0\,ft/s$

horizontal velocity $=$

$(75)\cos(30°)$ mph $= \frac{75\sqrt{3}}{2}$ mph $= 95.3\,ft/s$ (last time)

convert.



speed $= \sqrt{(95.3)^2 + (58)^2}$

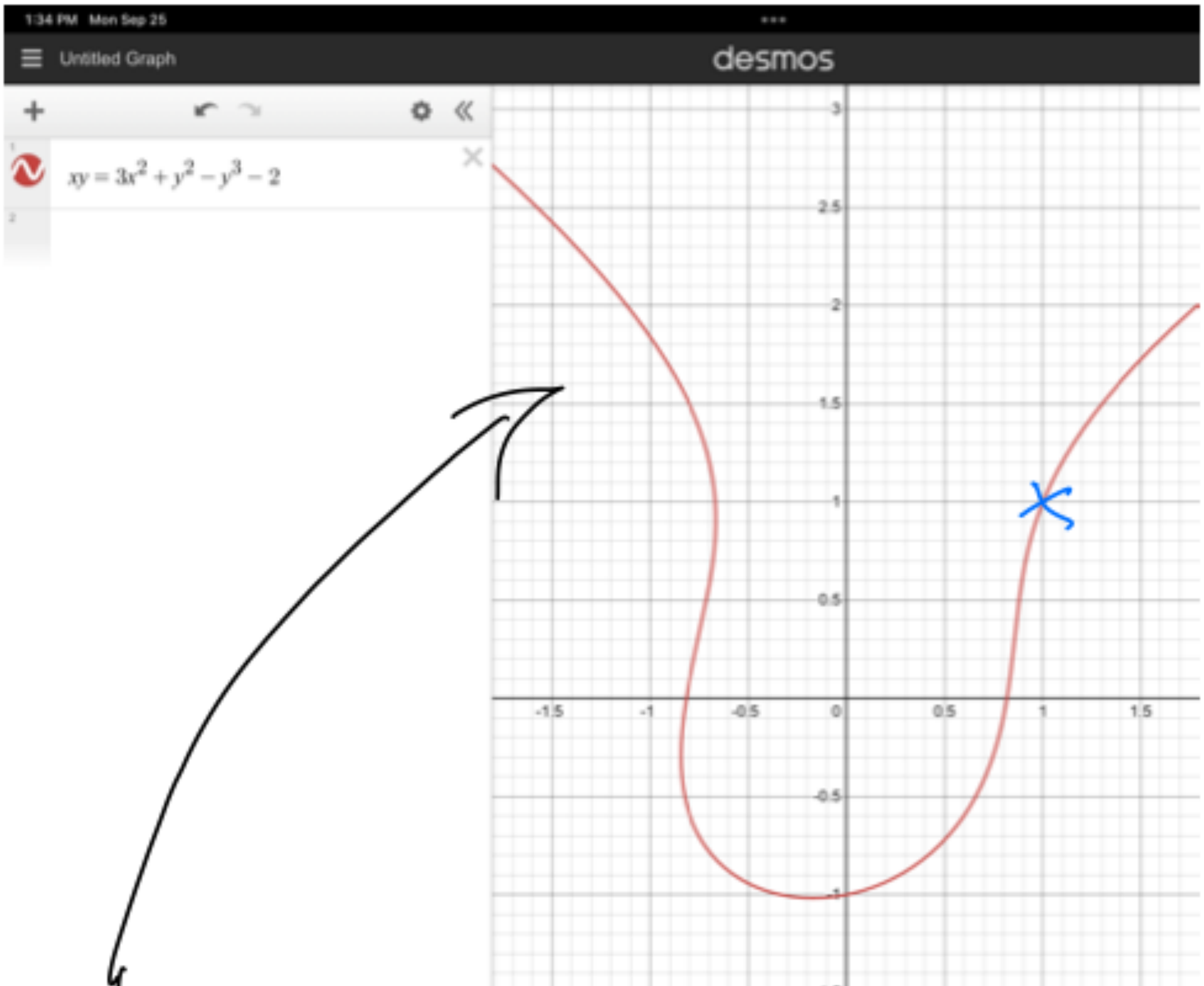
$= 111.56\,ft/s$

$= \boxed{112\,ft/s}$

Note $\frac{111.56\,ft}{s} \cdot \frac{mile}{5280\,ft} \cdot \frac{(60)^2 s}{hour} \approx \boxed{76.1\,mph}$

Finding derivatives for equations that are not functions:

Implicit Differentiation



This graph is not a function, but it's given by the equation

$$xy = 3x^2 + y^2 - y^3 - 2.$$

(1,1) is on the graph, since $x=1, y=1$ makes the equation true.

Questions: ① How would we find $\frac{dy}{dx}$ (ie slope of tangent line to the graph?)

② In particular, what is the equation of the tangent line to the graph at $(1,1)$?

③ At what points does this graph have a vertical tangent line?

TRICK:
- Pretend y is a function of x
- Compute the derivative of both sides of the equation
- Use chain rule when necessary.
- Solve for y'.

Solution: $xy = 3x^2 + y^2 - y^3 - 2$.

Derivative:

$$(x)'y + x(y)' = 6x + (y^2)' - (y^3)' - 0$$

$$\Rightarrow y + xy' = 6x + 2y \cdot y' - 3y^2 \cdot y'$$

$$\Rightarrow y - 6x = -xy' + 2yy' - 3y^2 y'$$

$$\Rightarrow y-6x = y'(-x+2y-3y^2)$$

$$\Rightarrow \boxed{\frac{y-6x}{-x+2y-3y^2} = y'} \quad (1)$$

$$\frac{dy}{dx} = y'$$

(2) Eqn of tangent line @ $(x,y)=(1,1)$.

$$\Rightarrow y' = \frac{1-6\cdot 1}{-1+2\cdot 1-3\cdot 1^2} = \frac{-5}{-2} = \boxed{\frac{5}{2}}$$

$$(y-y_0) = m(x-x_0)$$

$$y-1 = \frac{5}{2}(x-1) \Rightarrow y = \frac{5}{2}x - \frac{5}{2}+1$$

$$\boxed{y = \frac{5}{2}x - \frac{3}{2}}$$

(3) When does this graph have a vertical tangent line?

denom $=0$ $\quad -x+2y-3y^2=0$

$$x = 2y-3y^2$$

orig equation $\quad xy = 3x^2+y^2-y^3-2.$

$$(2y-3y^2)y = 3(2y-3y^2)^2+y^2-y^3-2$$

in theory, we would solve this for y.

<u>sagemath:</u> . . .